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FORCES IN THREE DIMENSIONS

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ABSTRACT

A rigid body is composed of many particles, the distance between the two particles remains constant, no matter what forces act on it. No perfectly rigid body exists in nature. Bodies change their sizes and shapes under the stress of great forces .When a rigid body is being acted upon by forces in different directions in space; we say that body is under the action of forces in the three dimensions.

If X, Y, Z be the components of a force R along x, y and z-axis respectively then

$R^2 = X^2 + y^2 + Z^2$.

Direction cosine ofline of action of R are $<\frac{X}{R}, \frac{Y}{R}, \frac{Z}{R}>$

Conversely if direction cosines of line of action of force Rare<l, m, n>then the components force R along the axes are lR, mR, nR respectively. Since a couple can be specified by a straight line drawn in a certain direction, therefore it is a vector quantity. Hence couples can be resolved or compounded by the parallelogram law in the same way as the forces.. If there are three component couples about the axis of *x*, *y* and z whose moments L, M, N are represented by OA, OB and OC respectively, then they compound into a single couple whose moment then $G^2 = L^2 + M^2 + N^2$.

Direction cosine of line of action of G are $<\frac{L}{G}, \frac{M}{G}, \frac{N}{G}>$

Conversely if direction cosines of line of action of force Gare< l, m, n>then the components of G along the axes are lG, mG, nG respectively.

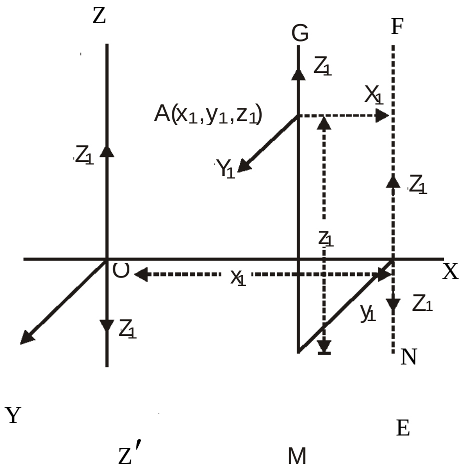
Any given system of forces acting at different points of a rigid body cans be reduced to a single force through any arbitrary chosen point and a couple whose axis passes through that point.

Any system of forces acting on a rigid body can be reduced to a single force together with a couple axis is along the direction of the forces.

INTRODUCTION

Any system of forces acting on a rigid body can be reduced in general to a force acting at an arbitrary chosen point of the body and a couple**.**

Take the arbitrary chosen point O as the origin and the three mutually perpendicular lines OX, OY, OZ through O as axes of co-ordinates.



Let one of the forces P of the system act at A (x_1, y_1, z_1) whose components parallel to the axes are X_1, Y_1, Z_1.

From A draw AM perpendicular on the plane XOY and from M draw MN perpendicular to OX.

∴ ON = x_1, NM = y_1 and MA = z_1

At N and O introduce two equal and opposite forces each equal and parallel to Z_1. These, being in equilibrium, do not affect the resultant of the system.

Now the forces Z_1 along NF and Z_1 along OZ′ form a couple of moment $- x_1Z_1$ in a plane perpendicular to the axis of y about OY. Again forces Z_1 along NE and MG form a couple of moment $y1Z1$ about OX.

Therefore, we find that the force Z_1 at A(x_1, y_1, z_1) is equivalent to

(i) an equal and parallel force Z_1 along the axis of z

(ii) a couple of moment y_1Z_1 about x-axis

(iii) a couple of moment – x_1Z_1 about y-axis.

Similarly, the component X_1 of the force at A(x_1, y_1, z_1) is equivalent to a single force X_1 at O along OX, a couple of moments z_1X_1 about OY and a couple of moment – y_1X_1 about OZ.

And the component Y_1 of the force at A (x_1, y_1, z_1) is equivalent to force Y_1 along OY, a couple of moment x_1 Y_1 about OZ and a couple of moment – z_1Y_1 about OX.

Combining the above three results, the components X_1, Y_1, Z_1 of the force at A(x_1, y_1, z_1) are together equivalent to the forces X_1, Y_1, Z_1 along OX, OY and OZ respectively and

a couple of moment (y_1Z_1– z_1Y_1) about OX

a couple of moment (z_1X_1– x_1Z_1) about OY

a couple of moment (x_1Y_1– y_1X_1) about OZ.

Similarly the force acting at another point A_1(x_2, y_2, z_2) with components X_2, Y_2, Z_2 can be replaced by forces acting along OX, OY and OZ and couples about these lines as axes.

Hence all the forces of the given system can be treated in this way so that the whole system of the forces is equivalent to

a force along OX = $X_1 + X_2 +$………………………………………..= Σ X_1 = X

a force along OY = $Y_1 + Y_2 +$………………………………………..= Σ Y_1 = Y

a force along OZ = $Z_1 + Z_2 +$………………………………………..= Σ Z_1 = Z

a couple of moment Σ (y_1Z_1– z_1Y_1) = L about OX

a couple of moment Σ (z_1X_1– x_1Z_1) = M about OY

anda couple of moment Σ (x_1Y_1– y_1X_1) = N about OZ.

The above three forces X, Y, Z are equivalent to a single force R acting through O, whereR^2 = $X^2 + y^2 + Z^2$.

Direction cosine of line of action of R are $\frac{X}{R}, \frac{Y}{R}, \frac{Z}{R}$

Also the above couples of moments L, M, N are together equivalent to a single couple ofmoment G, whose axis passes through O such that

$G^2 = L^2 + M^2 + N^2$.

Direction cosine of line of action of G are $\frac{L}{G}, \frac{M}{G}, \frac{N}{G}$

Hence the system of forces acting on a rigid body has been reduced to a single force R acting through an arbitrary chosen point O and a couple G whose axis passes through O.

Also any system of forces acting on a rigid body can be reduced to a single force together with a couple whose axis lies along the direction of the force.

Def^n**Wrench:** A single force R, together with a couple Gi.esystem (R, G) whose axis coincides with the line ofaction ofR is said to form a wrench.

Def^n**Poinsot's central axis:** The axis of single couple and the line of action ofR to which thesystem of force is reduced, is called the Poinsot's central axis or simply central axis.Central axis for a system of forces is unique.

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